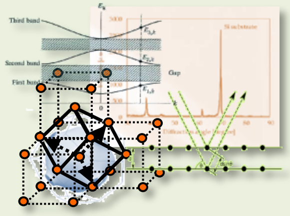
بسم الله الرحمن الرحيم

Solid State Physics Phys(471)

Lectures 19-22



ENERGY BANDS IN SOLIDS

THE EFFECTIVE MASS

Recall that in **1D**; $v = \frac{1}{\hbar}\frac{dE}{dk} \Rightarrow \boxed{\frac{dv}{dk} = \frac{1}{\hbar}\frac{d^2E}{\partial k^2}}$ **(11)**

When an electric field **ε** is applied to a crystal, the Bloch electron will undergo an acceleration **a** where;

$$a = \frac{dv}{dt} = \frac{dv}{dk}\frac{dk}{dt} \qquad \textbf{(12)}$$

The electric force ***F*** that caused this acceleration is given by

$$F = \hbar\frac{dk}{dt} \Rightarrow \boxed{\frac{dk}{dt} = \frac{F}{\hbar}} \qquad \textbf{(13)}$$

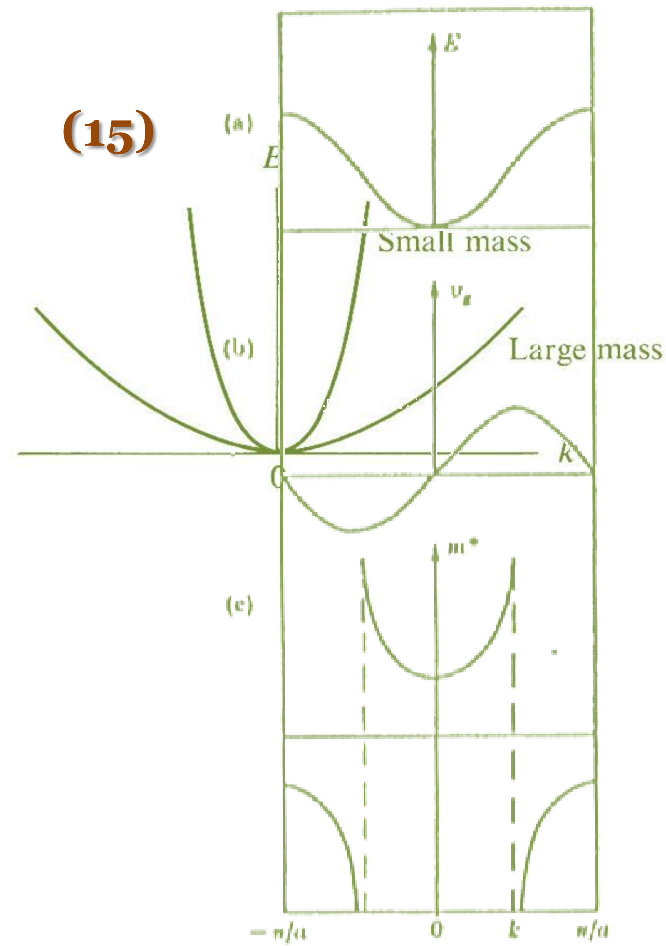
Substituting from (11) & (13) into (12) leads to;

$$a = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}F \qquad \textbf{(14)}$$

ENERGY BANDS IN SOLIDS

Eq. **(14)** has the same form as Newton's second law, hence one can define the electron ***effective mass*** as,

$$\boxed{m^* = \hbar^2 \Big/ \left(\frac{d^2E}{dk^2} \right)} \qquad \textbf{(15)}$$



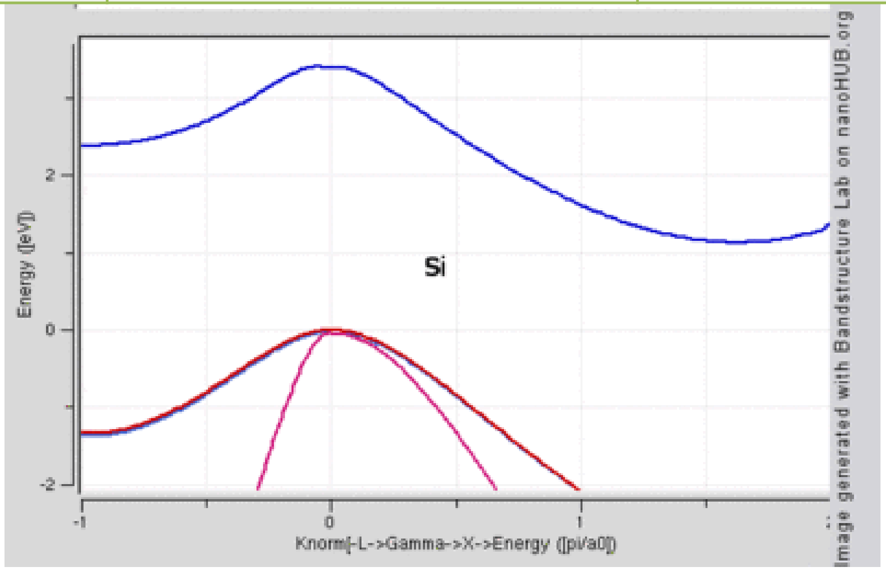
Note:

- When the curvature is large the mass is small, and the small curvature indicates a large mass.

- The effective mass ***m****** could be (+ve) or (-ve). Near the bottom of the band, the electron ***accelerates*** *and* ***m**** is ***positive***. But as the electron approaches the top of the band it will *decelerate*, and hence ***m**** is ***negative***.

ENERGY BANDS IN SOLIDS

Material	Electron effective mass	Hole effective mass
Si	$1.08\ m_e$	$0.56\ m_e$
Ge	$0.55\ m_e$	$0.37\ m_e$
GaAs	$0.067\ m_e$	$0.45\ m_e$



➤the effective mass m^* is related to the free electron mass m_e by the following relation:

$$\boxed{m^* = m_e \frac{F_{ext}}{F_{ext} + F_L}}$$

The Electrical conductivity

Recall: In the free electron model the electrical conductivity is given by;

$$\boxed{\sigma = \frac{ne^2\tau_F}{m^*}} \qquad \textbf{(16)}$$

Within the framework of **band theory**, a corresponding formula can be obtained as following:

- When an external electric field is applied, **FS** will be shifted a distance *dk* on the k-space. For **1D**;

$$\delta k_x = \frac{F}{\hbar}\delta t = -\frac{e\varepsilon}{\hbar}\delta t = \boxed{-\frac{e\varepsilon}{\hbar}\tau_F}$$

- The current density can be then written as;

$$J_x = -ev_{F,x}\,g(E_F)\,\delta E$$

$$= -ev_{F,x}\,g(E_F)\left[\frac{\partial E}{\partial k_x}\right]_{E_F}\delta k_x$$

But $\boxed{\dfrac{\partial E}{\partial k_x} = \hbar v_{F,x}}$

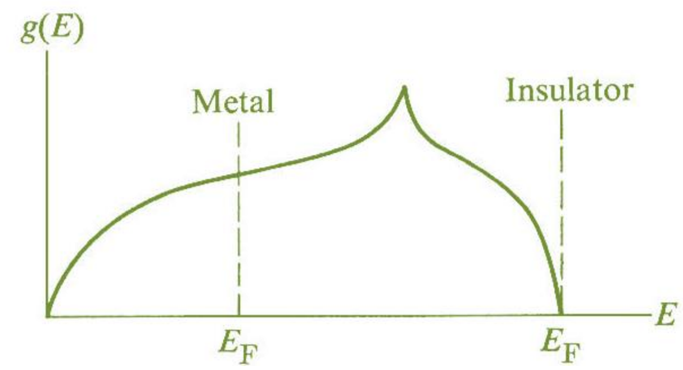
Which leads to; $J_x = e^2 v_{F,x}^2 \tau_F g(E_F)\varepsilon$

If FS is a sphere; $v_{F,x}^2 = \frac{1}{3} v_F^2$, hence;

$$J = \frac{1}{3} e^2 v_F^2 \tau_F g(E_F)\varepsilon$$

Therefore σ is;

$$\boxed{\sigma = \frac{1}{3} e^2 v_F^2 \tau_F g(E_F)} \qquad \textbf{(17)}$$



In **(17)** the predominant factor in determining σ is the density of state at FS $g(E_F)$, not the electron density n as **(16)** states.

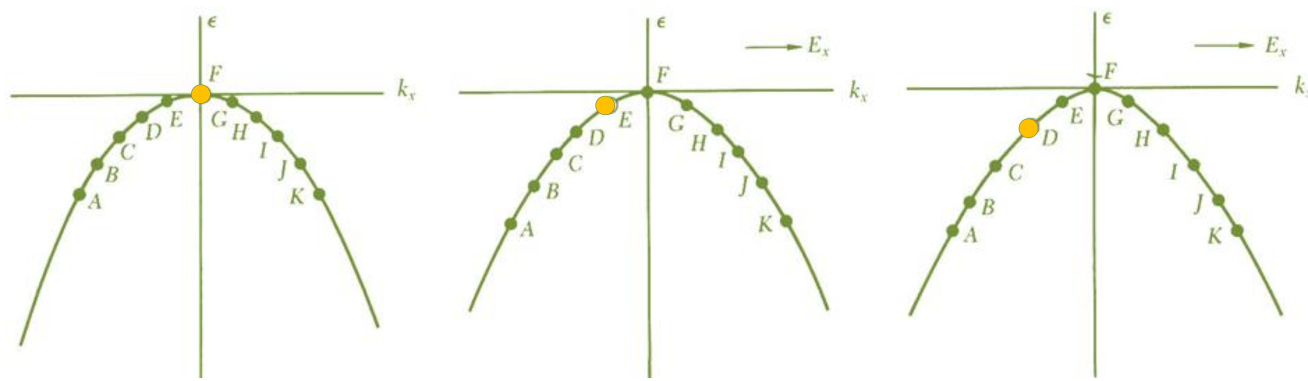
In fact **(16)** is an special case of **(17)** results when Fermi energy is taken as :

$$\boxed{E_F = \frac{\hbar^2 k_F^2}{2m^*}}$$

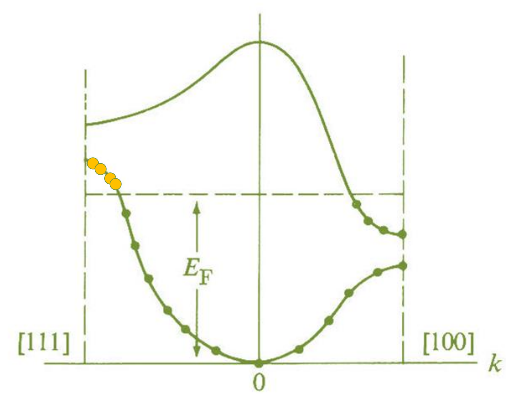
ENERGY BANDS IN SOLIDS

The Hall Effect

Define: a ***hole*** as one vacant state occurs in a totally full band.



When two bands overlap with each other, electrons will exist in the upper band and holes in lower.



ENERGY BANDS IN SOLIDS

The Hall constant expression for **a metal contains both electrons and holes** is;

$$\boxed{R = \frac{R_e \sigma_e^2 + R_h \sigma_h^2}{(\sigma_e + \sigma_h)^2}}$$

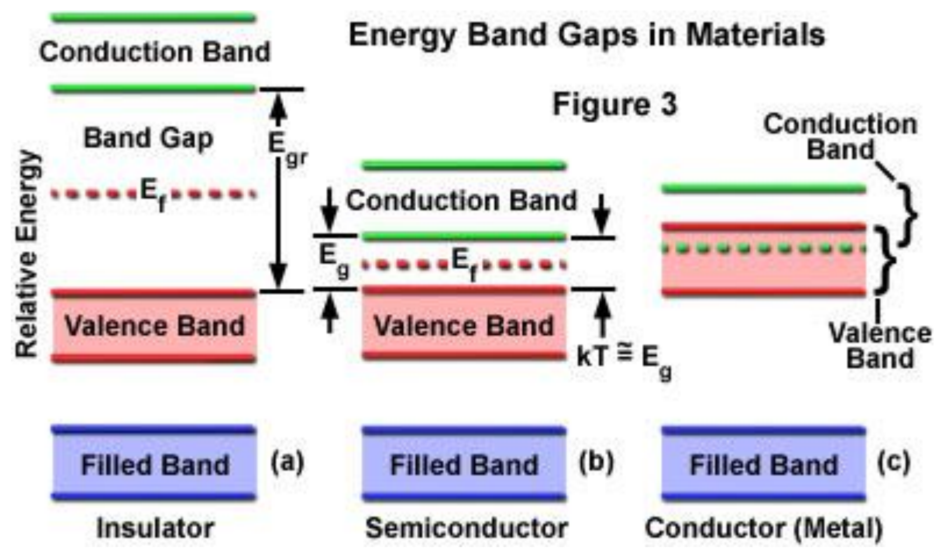
where R_h is the hole Hall constant given by,

$$\boxed{R_h = \frac{1}{n_h e}}$$

The total Hall constant R may be **(-ve)** or **(+ve)** depending on whether the contribution of the **electrons** or the **holes** dominates.

SEMICONDUCTORS

Semiconductors are materials whose electrical properties lie between Conductors and Insulators



Semiconductors can be classified as:

1- Intrinsic Semiconductors

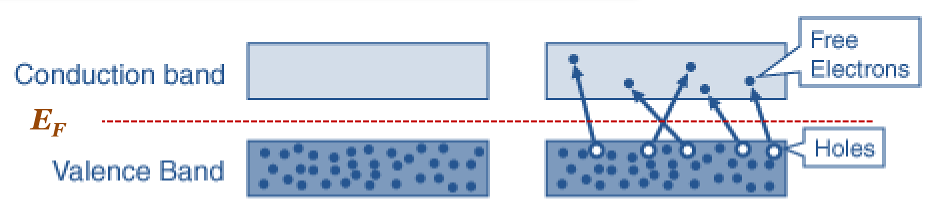
2- Extrinsic Semiconductors

SEMICONDUCTORS

1- Intrinsic Semiconductors :

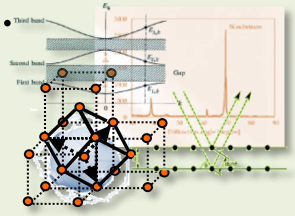
- The substance is pure, and hence the carrier concentration is an ***intrinsic*** property.
- The substance conducts current by both carriers electrons and holes.
- The concentration of electrons and the concentration of holes are equal **~ $10^{10}/cm^3$**.

$$\boxed{n = p = 2\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2}(m_e m_h)^{3/4}e^{-E_g/2kT}} \Rightarrow \boxed{n = p \propto e^{-E_g/2kT}}$$



The Fermi level lies at the middle of the energy gap, i.e.

$$\boxed{E_F = \tfrac{1}{2}E_g}$$



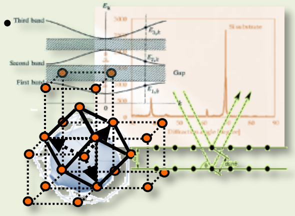
SEMICONDUCTORS

- The most common examples of intrinsic semiconductors are **silicon** and **germanium**. where

Material	Symbol	Type of Gap	Band gap (eV) @ 302K
Silicon	Si	indirect	**1.11**
Germanium	Ge	indirect	**0.67**

2- Extrinsic Semiconductors :

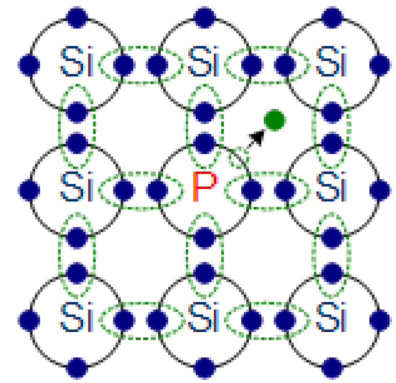
- The substance contains a large number of impurities which supply most of the carriers. Hence the carrier concentration is an *extrinsic property*.
- They conducts current by only one carriers electrons or holes.
- The carrier concentration is about $\sim 10^{15}/cm^3$. But by **heavy *doping*** one can get sample with concentration of $10^{18}/cm^3$.



SEMICONDUCTORS

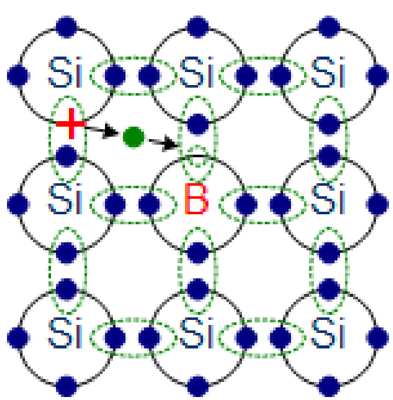
N-type Semiconductors

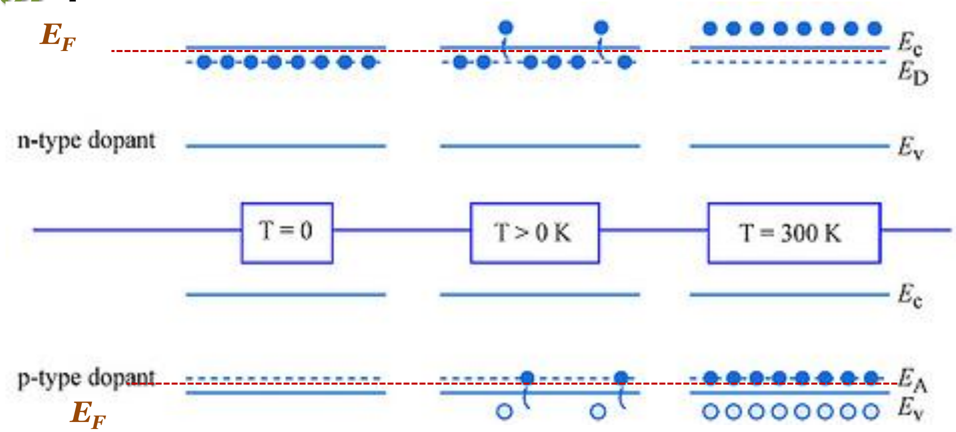
When a tetravalent sample (Si) is *doped* by a pentavalent atoms (P), each impurity atom will contribute *an electron* to the CB. Because of that these impurities are called ***donors*** and the substance is known as ***n-type Semiconductor.***

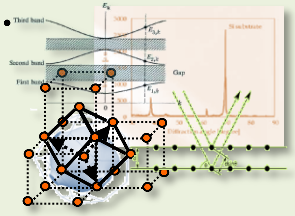


P-type Semiconductors

When a tetravalent sample (Si) is *doped* by a trivalent atoms (B), each impurity atom will contribute *a hole* to the VB. Because of that these impurities are called ***acceptors*** and the substance is known as ***p-type Semiconductor*** .

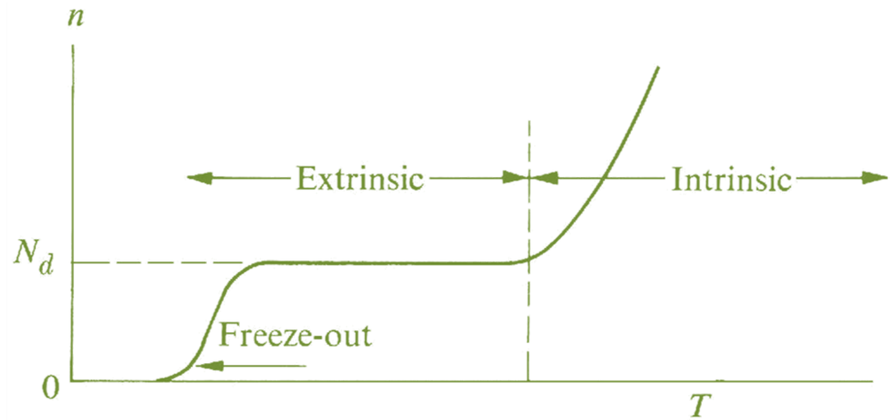






SEMICONDUCTORS

Fact **All semiconductors become intrinsic at high temperatures**



Extrinsic region :
Using the common doping rate **~ 10¹⁵/cm³** ,the number of carriers supplied by **impurities** at 300K is large enough to change the *intrinsic concentration* n_i.

- In this case :

$$\boxed{np = n_i^2}$$

- When $N_d >> N_a$ (**n-type**);

$$\boxed{n = N_d \;\Rightarrow\; p = \frac{n_i^2}{N_d}}$$

- When $N_a >> N_d$ (**p-type**);

$$\boxed{p = N_a \;\Rightarrow\; n = \frac{n_i^2}{N_a}}$$

Intrinsic region :
This region obtains when the impurity doping is so small . The concentration of electrons equals to the concentration of holes equals to what called *the intrinsic concentration* n_i ;
Which in the range of **~ 10¹⁰/cm³**

$$\boxed{n = p = n_i = 2\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2}(m_e m_h)^{3/4} e^{-E_g/2kT}}$$

$$\boxed{n_i \rangle\rangle (N_d - N_a)}$$

where N_d, N_a are the concentration of donors and acceptors respectively.

SEMICONDUCTORS

Electrical Conductivity and Mobility

- Assume an n-type semiconductor, using the free electron model the electrical conductivity is given by;

$$\boxed{\sigma_e = \frac{ne^2\tau_e}{m_e}}$$

- In semiconductors, transport characteristic is often described by ***mobility***, ***the ratio between the electron velocity and the applied field*** ,*i.e;*

$$\boxed{\mu_e = \frac{v_d}{e}} \quad \text{where} \quad \boxed{v_d = -\frac{e\tau}{m^*}\varepsilon} \Rightarrow \boxed{\mu_e = \frac{e\tau_e}{m_e}}$$

One can express the electrical conductivity in terms of mobility as;

$$\boxed{\sigma_e = ne\mu_e}$$

Total conductivity in a sample contains both carriers is

$$\boxed{\sigma = ne\mu_e + ne\mu_h}$$

SEMICONDUCTORS

Temperature dependence of Conductivity

In the intrinsic region: The conductivity is expressed by

$$\sigma = ne\mu_e + pe\mu_h$$

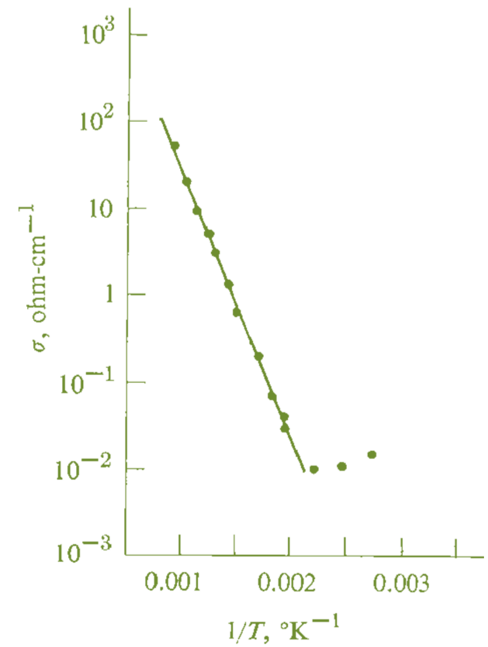
In this situation the concentration $n=p=n_i$ increases exponentially with temperature, thus;

$$\sigma = f(T)e^{-E_g/2kT}$$

where *f(T)* is a function which depends only weakly on the temperature (The function depends on the mobilities and effective masses of the particles.)

Taking the logarithms for both sides of the equation gives;

$$\log \sigma = Cons. - \frac{E_g}{2kT}$$



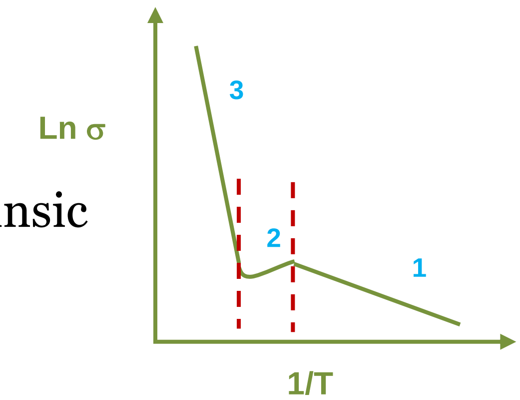
A plot of log σ versus 1/T should give a straight line with a **slope** of (- Eg/2k) that determines ***the energy gap*** of the material.

SEMICONDUCTORS

In the extrinsic region:

Region 1: in which conductivity occurs by impurities. Suppose that the substance is extrinsic **n-type**. The conductivity is

$$\boxed{\sigma_e = ne\mu_e}$$

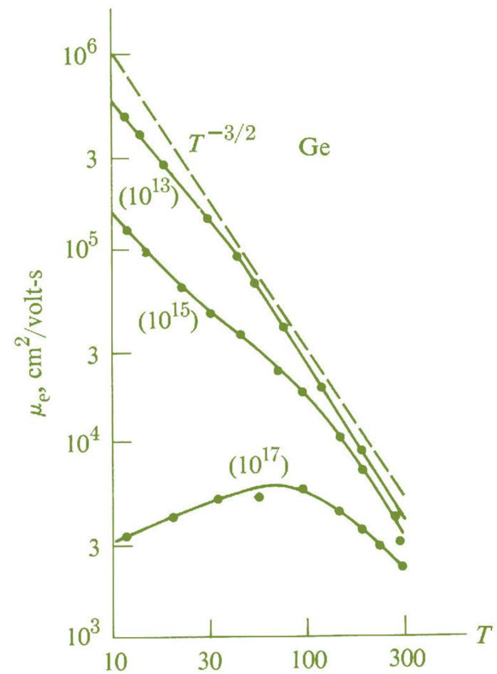


As T increases n increases **exponentially** and hence the conductivity σ. **I.e:**

$$\sigma \propto n \propto exp(-E_d/2kT).$$

Thus the **slope** of this region gives the ionization energy E_d of the semiconductor.

Region 2: A Saturation region, in which conductivity still occurs by impurities, but $n=N_d$ is constant. Hence **mobility** is the dominant factor.



As T increases μ **decreases** as $\mu_e \propto T^{-3/2}$ and hence the **conductivity** σ.

Hall Effect in Semiconductors

The Hall constant expression for a semiconductor contains both electrons and holes is;

$$R = \frac{R_e \sigma_e^2 + R_h \sigma_h^2}{(\sigma_e + \sigma_h)^2}$$

$$\boxed{R = \frac{p\mu_h^2 - n\mu_e^2}{e(n\mu_e + p\mu_h)^2}}$$

The total Hall constant R may be **(-ve)** or **(+ve)** depending on whether the contribution of the **electrons** or the **holes** dominates. It may vanish in semiconductors that reflect a high degree of symmetry.

SEMICONDUCTORS

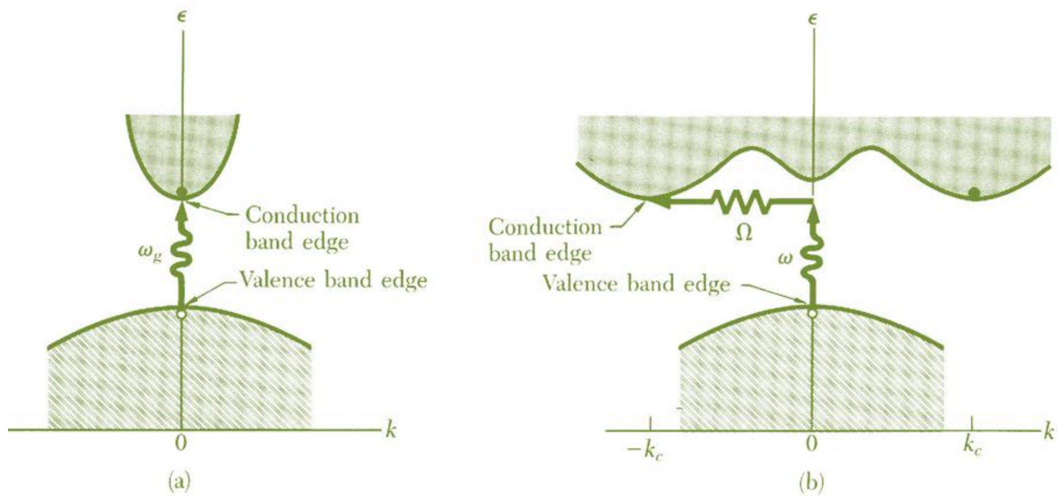
Direct and Indirect-Gap Semiconductors

the band gap of a semiconductor is always one of two types, a **direct band gap** or an **indirect band gap**.

If the **k-vectors** of the **minimal-energy** state in the conduction band, and the **maximal-energy** state in the valence band are;

the same, it is called a "**direct gap**".

If they are different, it is called an "**indirect gap**".



SEMICONDUCTORS

THE FUNDAMENTAL ABSORPTION

In fundamental absorption, **an electron absorbs a photon** (from the incident beam), **and jumps from the valence into the conduction band**. The photon energy must be equal to the energy gap, or larger. Therefore, the frequency must be

$$\nu \geq E_g / h$$

The frequency $\upsilon_o = E_g/h$ is referred to as ***the absorption edge***.

In the photon absorption process , the total energy and momentum of the electron-photon system must be conserved. Therefore

$$\boxed{E_f = E_i + h\nu \qquad \& \qquad k_f = k_i + q}$$

However, since the wave vector of the photon q is negligibly small. Thus, the momentum condition reduces to

$$\boxed{k_f = k_i}$$

This selection rule means that **only vertical transitions in k-space are allowed** between the valence and conduction bands.

SEMICONDUCTORS

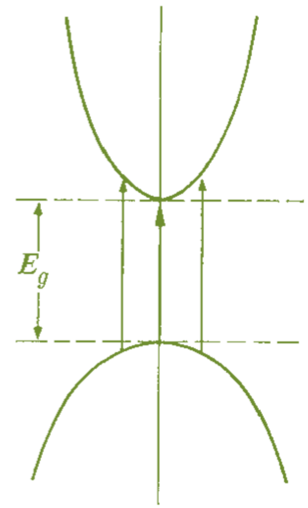
For Direct-Gap Semiconductors:

the absorption coefficient has the form

$$\boxed{\alpha_d = A(h\nu - E_g)^{1/2}}$$

where A is a constant involving the properties of the bands.
A useful application of these results is their use in measuring energy gaps in semiconductors. Thus E_g is directly related to the frequency edge, $E_g = h\upsilon_o$.

This is now the standard procedure for determining the gap, because of its accuracy and convenience. The optical method also reveals many more details about the band structure than the conductivity method.



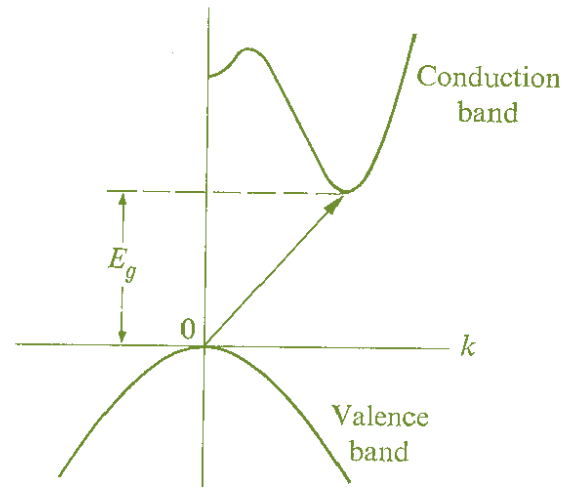
Since the energy gaps in semiconductors are small (1 eV or less) *the* ***fundamental edge*** usually occurs in the **infrared** region.

SEMICONDUCTORS

For Indirect-Gap Semiconductors:

In this case, the electron cannot make a direct transition from the top of the valence band to the bottom of the conduction band because this would violate the momentum selection rule.

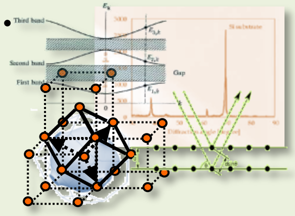
Such a transition may still take place, but as a **two-step process**. The electron absorbs both a **photon** and a **phonon** simultaneously. The photon supplies the needed energy, while the phonon supplies the required momentum.



The absorption coefficient in this case has the form;

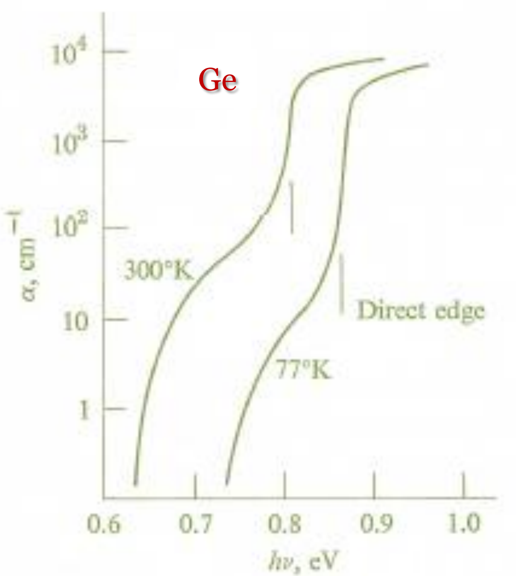
$$\boxed{\alpha_i = A^{`}(T)(h\nu - E_g)^2}$$

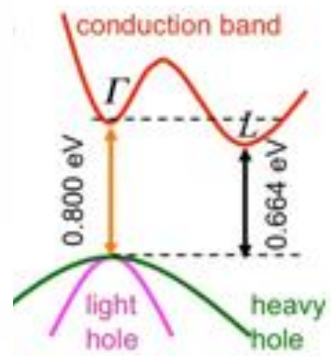
where *A'(T)* is a constant depends on temperature due to the phonon contribution to the process.



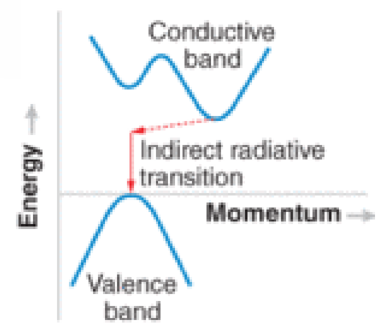
SEMICONDUCTORS

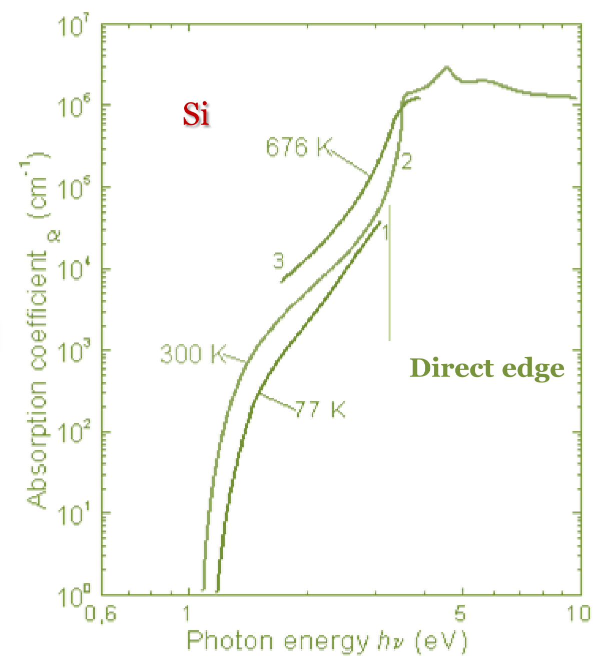
Note that α_i increases as the second power of $(h\nu - Eg)$, much **faster** than the half-power of this energy difference, as in the direct transition. So we may use the optical method to distinguish between direct- and indirect-gap semiconductors





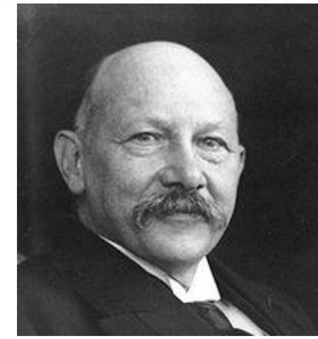
Indirect bandbap (Si)





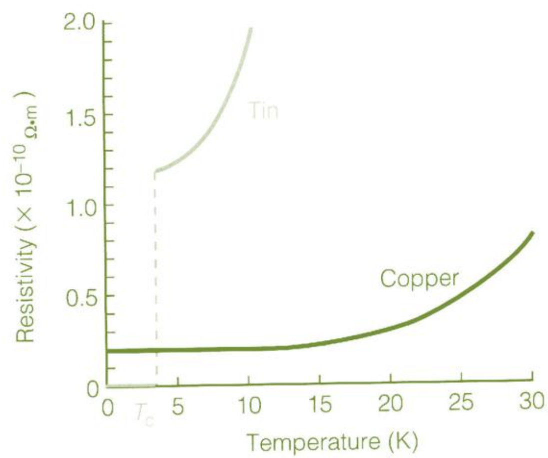
SUPERCONDUCTIVITY

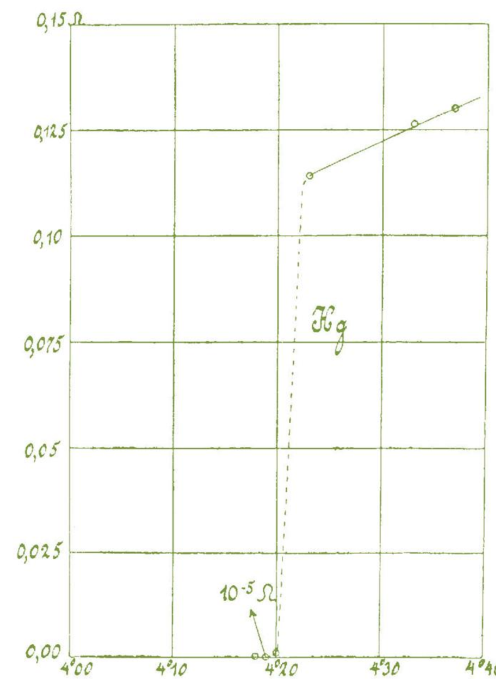
Superconductivity was discovered on **April 8, 1911** by **Heike Onnes**, who was studying the resistance of solid mercury at cryogenic temperatures using the recently-produced liquid helium as a refrigerant.



At the temperature of **4.2 K,** he observed that the resistance abruptly disappeared

In subsequent decades, superconductivity was observed in several other materials





What is a superconductor ?

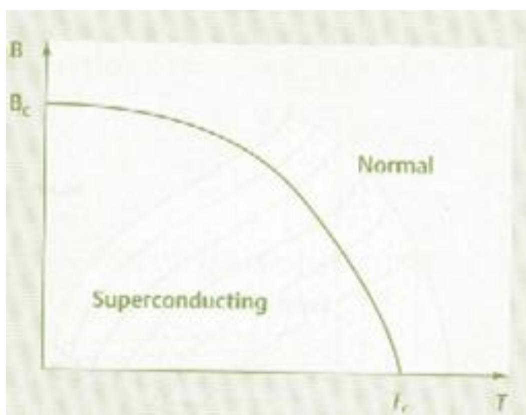
A superconductor: is a material that has zero electrical resistance when cooled below a particular transition temperature known as *the critical temperature* (T_c).

SUPERCONDUCTIVITY

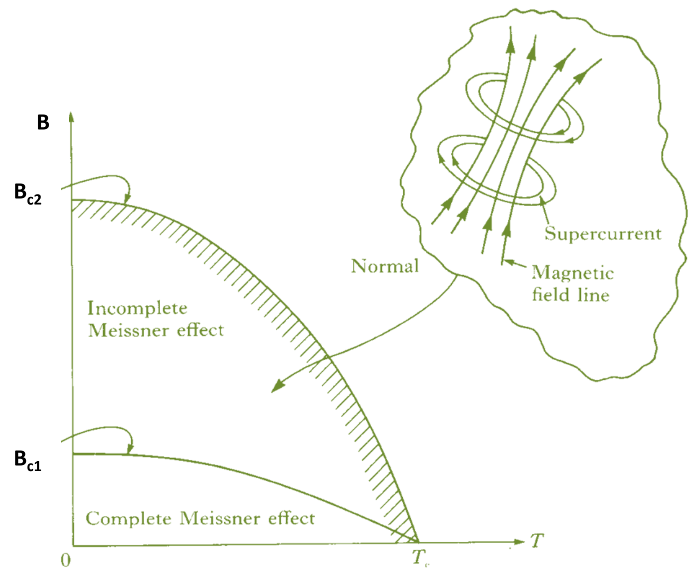
Main characteristics:

1- Zero resistance: Under particular values of the magnetic field (B_c) and the current density (J_c) , the superconductor material show a zero resistance.

$$B_T = B_c\left[1-\left(\frac{T}{T_c}\right)^2\right]$$



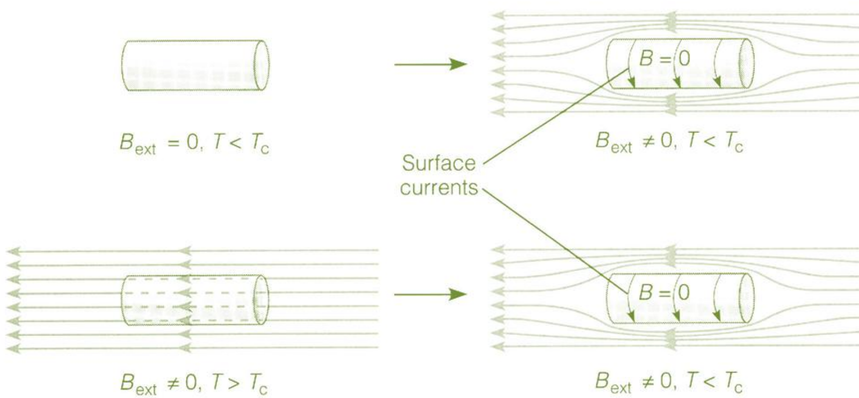
Type I superconductor

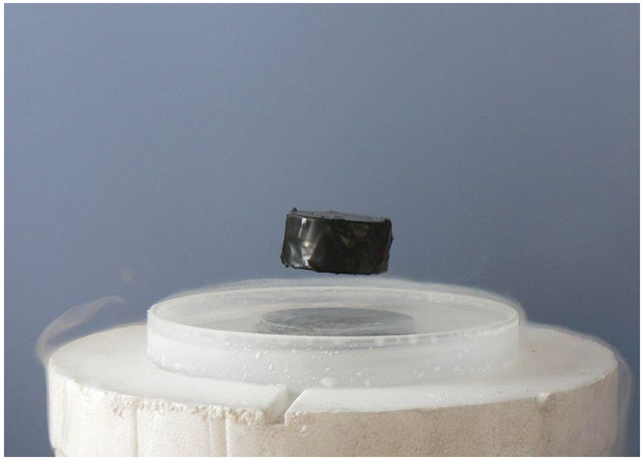


Type II superconductor

SUPERCONDUCTIVITY

2- Meissner effect : superconductors expel magnetic flux such that $B = 0$ inside.



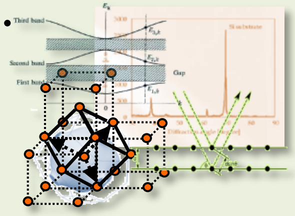


Important Notes:

1- Good conductors at **RT** do not superconduct at low temperatures.

2- T_c of two different isotopes are found to be proportional to $1/\sqrt{M}$ where M is the atomic mass.

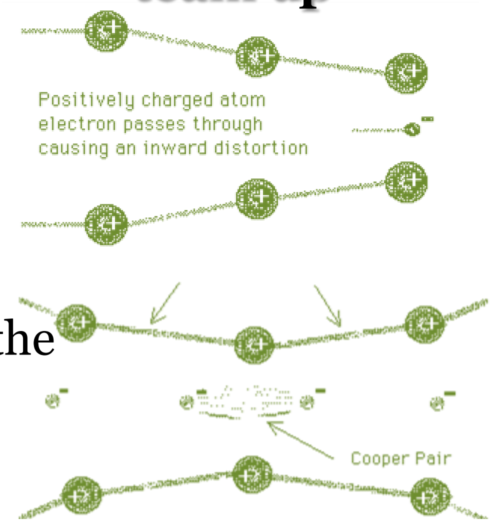
These ideas were central to the superconductivity's theory which was formulated in **1957** by John **B**ardeen, Leon **C**ooper, and Robert **S**chrieffer, and known as (**BCS**) **theory**.



SUPERCONDUCTIVITY

BCS THEORY

The basic assumption in **BCS** theory is that electrons **team up** in pairs called ***Cooper pairs***.



What are Cooper pairs?

Below T_c electrons form pairs mediated by **phonons** known as ***Cooper pairs***.
This pair is only formed if the 2nd electron has the same speed as the 1st but in opposite direction.

BCS theory:

Pairs of electrons can behave very differently from single electrons which are **fermions** and must obey the Pauli exclusion principle. The pairs of electrons act more **like bosons** which can condense into the same energy level. The electron pairs have a slightly lower energy and leave an **energy gap** above them on the order of **.001 eV** which inhibits the kind of collisions which lead to ordinary resistivity. For temperatures such that the thermal energy is less than the band gap, the material shows zero resistivity.

SUPERCONDUCTIVITY

Are Cooper pairs really exist?

BCS theory predicts that the **energy gap** is given by;

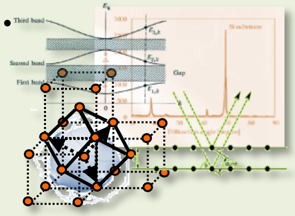
$$E_g = 3.53\, kT_c$$

The existence of such gap can be examined by shining a light onto the sample with a frequency ;

$$\nu \geq E_g / h$$

If the photon is absorbed then the gap exist. The absorbed photon will then destroy the electron pair and the material becomes normal.

Bardeen, **C**ooper, and **S**chrieffer received the Nobel Prize in **1972** for the development of the theory of superconductivity.

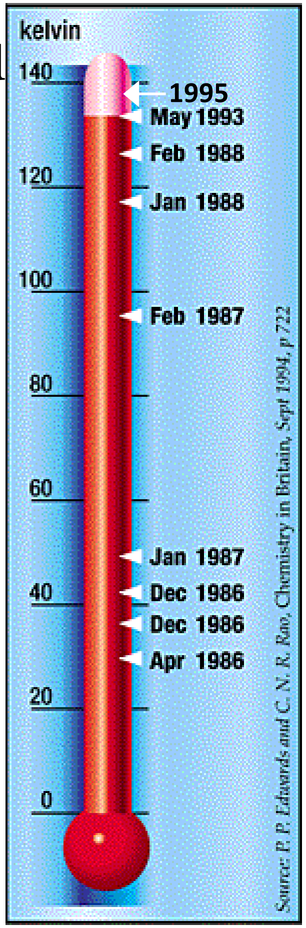


SUPERCONDUCTIVITY

High T_c superconductors or (HTS):

High-T_c are materials that have a superconducting transition temperature (T_c) above **30 K**. From **1960** to **1980**, **30 K** was thought to be the highest theoretically possible T_c.

the highest record is T_c ~ **138K**



The first high-T_c superconductor was discovered in **1986** by **IBM** Researchers **Karl Müller** and **Johannes Bednorz**, for which they were awarded the Nobel Prize in Physics in **1987**.

The question of how superconductivity arises in high-temperature superconductors is one of the major unsolved problems of theoretical condensed matter physics as of 2010.

SUPERCONDUCTIVITY

Applications

- **Transmission cables** that carry current without energy losses will increase the capacity of the transmission system, saving money, space, and energy.
- **Motors and Generators** made with superconducting wires will be smaller and more efficient
- **Magnetic resonance imaging (MRI) machines** enhance medical diagnostics by imaging internal organs. MRIs, which currently are made with low-temperature superconductors, will be smaller and less expensive when made with HTS.
- **Maglev trains** seem to float on air as a result of using superconducting magnets. The newest prototype may exceed **547 Km/hr**.

SUPERCONDUCTIVITY

➢ Superconducting magnets

The simplest, and perhaps most clear, application is to use the supercurrent to generate an intense **magnetic field**.

- Let us first consider an electromagnet made from a normal metal wire. The magnetic field produced by a long solenoid is given by

  where n is the number of turns per unit length and I is the current in the wire.

- **For example**, the maximum current density for **copper** is 400 A cm^{-2}. Thus, an electromagnet formed by winding 150 turns per meter using copper wire of diameter 3 mm will give maximum current equals to:

$$I_{max} = J_{max}\,\pi r^2 = (400\times10^4\ \mathrm{A\,m^{-2}})(3.14)(1.5\times10^{-3}\ \mathrm{m})^2 = 28.3\ \mathrm{A}$$

Such current will produce maximum field of;

$$B_{max} = = \mu_0\, n\, I_{max} = \mathbf{5.34\times10^{-3}\ T}$$

This is too small for most practical applications.

SUPERCONDUCTIVITY

- However, by **placing an iron core** within the solenoid, this magnetic field can be enhanced to about 2T. The main disadvantage of this arrangement is that the iron core is extremely **heavy** and **uncomfortable**.
- **Let us now consider a similar structure using a *superconducting* wire**.
- In this case the maximum current is determined by the ***critical current density***. Since it is common to achieve critical current densities of 10^7 A cm^{-2}, it is possible to produce very large magnetic fields (of **few hundred tesla**) with a superconducting solenoid.
- An additional advantage is that **no iron core** is required in this case. This dramatically reduces the size and weight of the electromagnet and so opens up a whole new range of applications which cannot be performed using large and heavy conventional electromagnets.

SUPERCONDUCTIVITY

SQUID magnetometers:

SQUID is an acronym for (**S**uperconducting **QU**antum **I**nterference **D**evice) magnetometer which is capable of measuring extremely small magnetic fields. It has already found applications in such various areas as **medicine** (measuring the small magnetic fields produced by activity in the brain), **geology** (detecting changes in the Earth's magnetic field due to the presence of oil or other mineral deposits) and **particle physics** (searching for quarks and other exotic particles).